Find the slope of the tangent line at the point $(-1, 1)$ on the curve $(x^2y^3 - y^2 + 1)^2 = 2 - y^3$.

SCORE: ____ / 5 PTS

$2(x^2y^3 - y^2 + 1)(2xy^3 + 3x^2y^2\frac{dy}{dx} - 2y\frac{dy}{dx}) = -3y^2\frac{dy}{dx}$

@ $(-1,1)$: (1) ($\frac{1}{2}$) (1) ($\frac{1}{2}$) ($\frac{1}{2}$)

$2(1-1+1)(-2 + 3\frac{dy}{dx}\Big|_{(-1,1)} - 2\frac{dy}{dx}\Big|_{(-1,1)}) = -3\frac{dy}{dx}\Big|_{(-1,1)}$ ($\frac{1}{2}$)

$2(-2 + \frac{dy}{dx}\Big|_{(-1,1)}) = -3\frac{dy}{dx}\Big|_{(-1,1)}$

$-4 + 2\frac{dy}{dx}\Big|_{(-1,1)} = -3\frac{dy}{dx}\Big|_{(-1,1)}$

$-4 = -5\frac{dy}{dx}\Big|_{(-1,1)}$

$\frac{dy}{dx}\Big|_{(-1,1)} = \frac{4}{5}$ (1)

Prove the derivative of $\tan^{-1} x$ using implicit differentiation.

SCORE: ____ / 4 PTS

$y = \tan^{-1} x$

(1) $x = \tan y$

(1) $1 = \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$

(1/2) (1) (1/2)

Find the following derivatives. Simplify all answers appropriately. (1) POINT EACH ITEM UNLESS OTHERWISE NOTED **SCORE: ____ / 21 PTS**

[a] $\frac{d}{dt}\cos^{-1}(t+\cos^2 t)$

$$= -\frac{1}{\sqrt{1-(t+\cos^2 t)^2}}\cdot(1+2\cos t(-\sin t))$$

$$= -\frac{1-2\cos t\sin t}{\sqrt{1-(t+\cos^2 t)^2}}$$ (2)

[b] $\frac{d}{dx}x^{\sec^2 x}$

$y = x^{\sec^2 x}$

$\ln y = \sec^2 x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\sec x(\sec x\tan x)\ln x + (\sec^2 x)\frac{1}{x}$ (2)

$\frac{dy}{dx} = y(\sec^2 x)(2\tan x\ln x + \frac{1}{x})$

$= x^{\sec^2 x}\sec^2 x(2\tan x\ln x + \frac{1}{x})$

$= x^{\sec^2 x - 1}\sec^2 x(2x\tan x\ln x + 1)$

$= x^{\tan^2 x}\sec^2 x(1+2x\tan x\ln x)$

BONUS (1)

[c] $\frac{d}{d\theta}\ln\frac{\sqrt[4]{\sin\theta}}{(1+\cot\theta)^5}$

$= \frac{d}{d\theta}(\frac{1}{4}\ln(\sin\theta) - 5\ln(1+\cot\theta))$

$= \frac{1}{4}\frac{1}{\sin\theta}\cos\theta - 5\frac{1}{1+\cot\theta}(-\csc^2\theta)$ (2)

$= \frac{1}{4}\cot\theta + \frac{5\csc^2\theta}{1+\cot\theta}$ (2)

[d] $\frac{d}{dy}\sin^{-1}\frac{y}{1-y}$

$= \frac{1}{\sqrt{1-(\frac{y}{1-y})^2}}\cdot\frac{1(1-y)-y(-1)}{(1-y)^2}$

$= \frac{1}{\sqrt{\frac{1-2y+y^2-y^2}{(1-y)^2}}}\cdot\frac{1}{(1-y)^2}$ (2)

$= \frac{1}{\frac{\sqrt{1-2y}}{|1-y|}}\cdot\frac{1}{(1-y)^2}$

$= \frac{1}{|1-y|\sqrt{1-2y}}$